NOTES

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IGCSE MATHEMATICS SUMMARY

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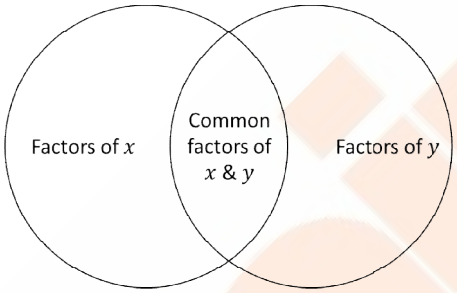
1. NUMBER

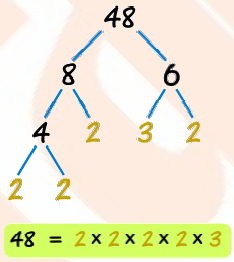
- **Natural numbers:**
  - used for counting purposes
  - all possible rational &irrational numbers
- **Integer:** a whole number
- **Prime numbers:**
  - divisible only by itself and one
  - 1 is not a prime number
- **Rational numbers:** can be written as a fraction
- **Irrational numbers:** cannot be written as a fraction e.g.π
- **Cube numbers:** made from multiplying a rational number to itself twice.
- **Reciprocals:** A number made by raising a rational number to -1, or 1 over that number

1.1 HCF and LCM

- **Highest Common Factor and Lowest Common Multiple:**



  - HCF = product of common factors of x and y
  - LCM = product of all items in Venn diagram
- **Prime Factorization:** finding which prime numbers
  - multiply together to make the original number

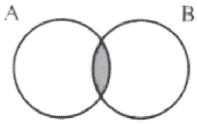


$48 = 2 \times 2 \times 2 \times 2 \times 3$

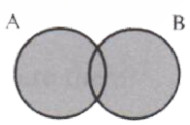
1.2 Sets

- Definition of sets e.g.
  - $A = \{x: x \text{ is a natural number}\}$
  - $B = \{(x, y): y = mx + c\}$
  - $C = \{x: a \leq x \leq b\}$
  - $D = \{a, b, c, \dots\}$

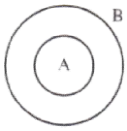
Set representations:



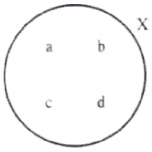
$A \cap B$ is shaded



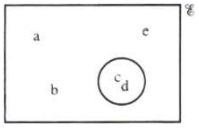
$A \cup B$ is shaded



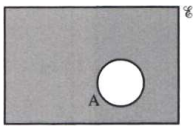
$\subset$ 'is a subset of'



$b \in X$



$\mathscr{E}$ = {a, b, c, d, e}



A' is shaded

Notation:

- $n(A)$ = no. of elements in A
- $\in$ = …is an element of…
- $\notin$ = …is not an element of…
- A' = compliment of set A
- $\emptyset$ or $\{\ \}$ = empty set
- $\mathscr{E}$ = Universal set
- $A \cup B$ = union of A and B
- $A \cap B$ = intersection of A and B
- $A \subseteq B$ = A is a subset of B
- $A \subset B$ = A is a proper subset of B
- $A \nsubseteq B$ = A is not a subset of B
- $A \not\subset B$ = A is not a proper subset of B

1.3 Indices

Standard form:

- $10^4 = 10000$
- $10^3 = 1000$
- $10^2 = 100$
- $10^1 = 10$
- $10^0 = 1$

$10^{-1} = 0.1$
$10^{-2} = 0.01$
$10^{-3} = 0.001$
$10^{-4} = 0.0001$
$10^{-5} = 0.00001$

Limits of accuracy:

- The degree of rounding of a number
  - E.g. 2.1 to 1 d.p $2.05 \leq x < 2.15$
  - Finding limits when adding/multiplying: add/multiply respective limits of values
  - Finding maximum value possible when dividing/subtracting: max value divided by/minus min value
  - Finding minimum value possible when dividing/subtracting: min value divided by/minus max value

1.4 Ratio & Proportion

- **Ratio:** used to describe a fraction
  - e.g. 3 : 1
- **Foreign exchange:** money changed from one currency to another using proportion
  - E.g. Convert $22.50 to Dinars

$$\$1 : 0.30KD$$
$$\$22.50 : 6.75KD$$

- **Map scales:** using proportion to work out map scales
  - 1km = 1000m
  - 1m = 100cm
  - 1cm = 10mm
- **Direct variation:** y is proportional to x

$$y \propto x \qquad y = kx$$

- **Inverse variation:** y is inversely proportional to x

$$y \propto \frac{1}{x} \qquad y = \frac{k}{x}$$

1.5 Percentages

- **Percentage:**
  - Convenient way of expressing fractions
  - Percent means per 100
- **Percentage increase or decrease:**

$$Percentage\ increase = \frac{Actual\ Change}{Original\ Amount} \times 100$$

- **Simple interest:**

$$I = \frac{PRT}{100}$$
$$P = Principal \quad R = Rate\ of\ Interest$$
$$T = Period\ of\ Time$$

- **Compound interest:**

$$A = P\left(1 + \frac{R}{100}\right)^n$$
$$P = Principal \quad R = Rate\ of\ Interest$$
$$n = Period\ of\ Time$$

1.6 Speed, Distance & Time

$$Speed = \frac{Distance}{Time}$$
$$Average\ Speed = \frac{Total\ Distance}{Total\ Time}$$

- **Units of speed:** km/hr or m/s
- **Units of distance:** km or m
- **Units of time:** hr or sec

$$km/hr \times \frac{5}{18} = m/sec$$
$$m/sec \times \frac{18}{5} = km/hr$$

2. Algebra & Graphs

2.1 Factorisation

- **Common factors:**

$$3x^2 + 6x$$
$$3x(x + 2)$$

- **Difference of two squares:**

$$25 - x^2$$
$$(5 + x)(5 - x)$$

- **Group factorization:**

$$4d + ac + ad + 4c$$
$$4(d + c) + a(c + d)$$
$$(4 + a)(c + d)$$

- **Trinomial:**

$$x^2 + 14x + 24$$
$$x^2 + 12x + 2x + 24$$
$$x(x + 12) + 2(x + 12)$$
$$(x + 2)(x + 12)$$

2.2 Quadratic Factorization

- **General equation:**

$$ax^2 + bx + c = 0$$

- **Solve quadratics by:**
  - Trinomial factorization
  - Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- When question says, "give your answer to two decimal places", **use formula!**
- Derivation of the Quadratic Formula is the same as saying "Make x the subject in $ax^2 + bx + c = 0$"→

$$ax^2 + bx + c = 0$$

Factorize a out

$$a\left(x^2 + \frac{b}{a}x\right) + c = 0$$

Complete the Square

$$a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right) + c = 0$$
$$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c = 0$$
$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a}$$
$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$
$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$
$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

Note: $4a^2$ is a square number

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- **Standardized form:**
  - $y = ax^2 + bx + c$
- **Complete Square form:**
  - $y = (x + a)^2 + b$ (Where axis of symmetry is $x = -a$)
  - To find turning point of quadratic equation, complete the square, then the turning point is: $(-a,\ b)$
- **Ways to solve Quadratic equation:**
  - Graphing Method
  - Factorizing
  - Quadratic Formula
  - Complete the Square

- **Graphing Method –** Graph the equation, see where the it touches the x-axis

- **Factorizing**

e.g. $x^2 - x - 6 = 0$

$$x^2 - x - 6 = 0$$
$$(x - 3)(x + 2) = 0$$
$$x_1 = 3$$
$$x_2 = -2$$

- **Quadratic Formula**

e.g. $x^2 - x - 6 = 0$

Where $a = 1, b = -1, c = -6$

Plug the numbers in the Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Therefore:

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-6)}}{2(1)}$$

$$x_1 = 3$$
$$x_2 = -2$$

-

- Complete the Square

e.g. $x^2 + 10x + 5 = 0$

(**WARNING!** Coefficient of x^2 Must be 1 for this to work)

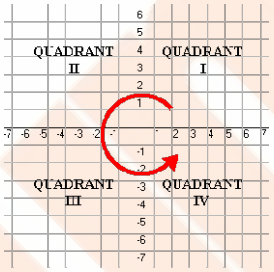
$$x^2 + 10x + 5 = 0$$
$$(x + 5)^2 - 5^2 + 5 = 0$$
$$(x + 5)^2 - 20 = 0$$
$$(x + 5)^2 = 20$$
$$x + 5 = \pm\sqrt{20}$$
$$x = -5 \pm \sqrt{20}$$

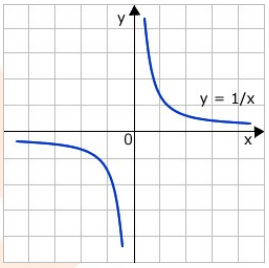
Answer is: $x_1 = -5 + \sqrt{20}$ and $x_2 = -5 - \sqrt{20}$

2.3 Reciprocal Graphs (Hyperbola)

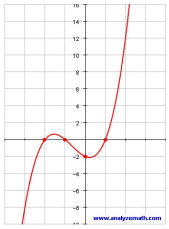
- **Standardized Form:**
  - $y = \frac{a}{x}$

If a is **Positive**:	If a is **Negative**:
The Line will be in the **1st&3rd** Quadrant	The Line will be in the **2nd&4th** Quadrant





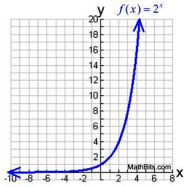
2.4 Cubic Equation



- **Standardized Form:**
  - $y = ax^3 + bx^2 + cx + d$
- **Properties:**
  - Highest Exponent of x is 3
  - Has a maximum of 2 turning points

  Turning points are points after which a graph changes its gradient's sign, therefore changing direction between up or down

2.5 Exponential Graphs

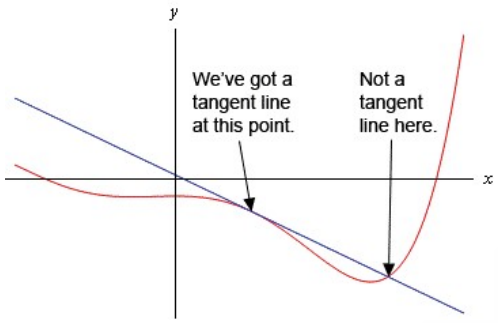


- **Standardized form:**
  - $y = a(b)^x$
- **Properties:**
  - a is the y-intercept
  - Asymptotes are lines that a curve approaches, but never touches because the curve continues to infinity, in this case the y-axis
  - b is the rate of growth
  - When $0 < b < 1$, the graph will go downwards from left to right

2.6 Gradient of a Curve

- **By drawing tangents**
  - In a straight line, gradient is constant
  - Curves have varying gradients throughout the graph

    To find the gradient at a point:
    1. Draw the graph
    2. Draw a tangent at the point in the graph, ensuring it only touches the graph at that point (Use a ruler)
    3. Find the gradient of the tangent



- **Using differentiation**
  - $\frac{dy}{dx}$ gives you the gradient of the curve at any point in terms of x
  - When $y = x^n$, $\frac{dy}{dx} = nx^{n-1}$
  - Stationary/ turning point: $\frac{dy}{dx} = 0$
  - 1st Derivative = $\frac{dy}{dx} = f'(x)$
  - 2nd Derivative = $\frac{d^2y}{dx^2} = f''(x)$
  - To determine if stationary point is maximum or minimum:
    - Use 2nd derivative
      - Maximum point: $\frac{d^2y}{dx^2} < 0$
      - Minimum point: $\frac{d^2y}{dx^2} > 0$
    - Use gradients around the point
      - Input x values slightly above and below stationary point and calculate gradient

2.7 Simultaneous Equations

- Can be solved either by substitution or elimination
- Generally solved by substitution as follows:
  - Step 1: obtain an equation in one unknown and solve this equation
  - Step 2: substitute the results from step 1 into linear equation to find the other unknown
- The points of intersection of two graphs are given by the solution of their simultaneous equations

2.8 Inequalities

- Solve like equations
- Multiplying or dividing by negative ⇒ switch sign

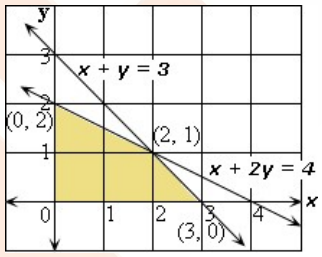
$$\frac{y}{-3} \geq -7$$
$$y \leq -7 \times -3$$
$$y \leq 21$$

- When two inequalities present, split into two

$$x < 3x - 1 < 2x + 7$$
$$x < 3x - 1 \qquad 3x - 1 < 2x + 7$$
$$x > -\frac{1}{2} \qquad x < 8$$

2.9 Linear Programming

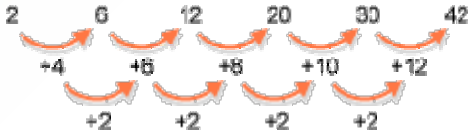
- For strict inequalities $(<, >)$ use broken line
- For non-strict inequalities $(\leq, \geq)$ use solid line
- Steps to solve:
  - Interpret $y = mx + c$
  - Draw straight line graphs
  - Shade
  - Solve



2.10 Sequences

- **Linear sequences:** Find common difference e.g. 3, then multiply by n and work out what needs to be added
- **Quadratic sequences:**
  - Format: $an^2 + bn + c$

$a + b + c =$
$3a + b =$
$2a =$



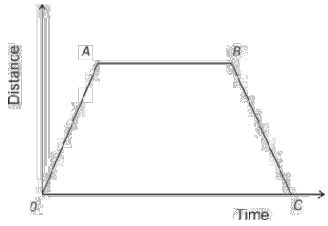
  - Work out the values and then place into formula to work out nth term formula

- **Geometric progression:** sequence where term has been multiplied by a constant to form next term

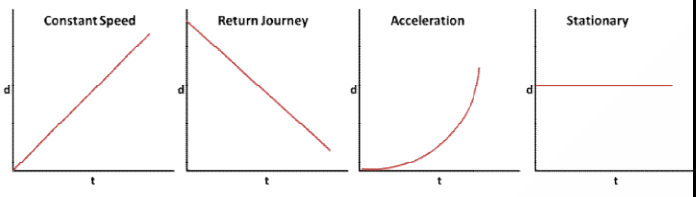
$$nth\ term\ of\ G.P. = ar^{(n-1)}$$

  - a = 1st term r = common difference

2.11 Distance-Time Graphs

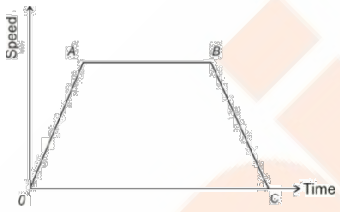


- From O to A: Uniform speed
- From B to C: Uniform speed (return journey)
- From A to B: Stationery (speed = 0)

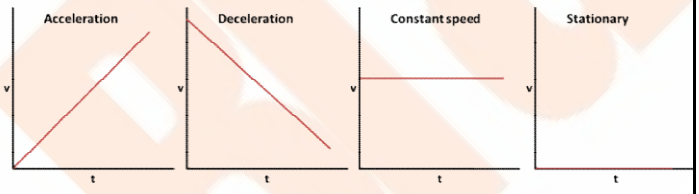


- Gradient = speed

2.12 Speed-Time Graphs



- From O to A: Uniform speed
- From A to B: Constant speed (acceleration = 0)
- From B to C:Uniform deceleration / retardation



- Area under a graph = distance travelled.
- Gradient = acceleration.
- If the acceleration is negative, it is called deceleration or retardation. (moving body is slowing down.)

2.13 Functions

- **Function notation:**
  - $f: x \to 2x - 1$
  - Function f such that x maps onto $2x - 1$
- **Composite function:** Given two functions $f(x)$ and $g(x)$, the composite function of f and g is the function which maps x onto $f\big(g(x)\big)$
- $f(2)$
  - Substitute $x = 2$ and solve for $f(x)$
- $fg(x)$
  - Substitute $x = g(x)$
- $f^{-1}(x)$
  - Let $y = f(x)$ and make x the subject

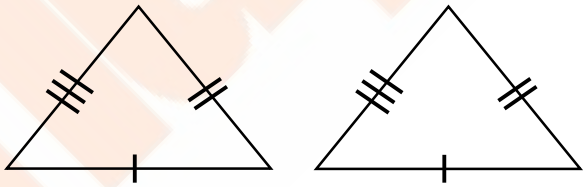
3. Geometry

3.1 Similarity

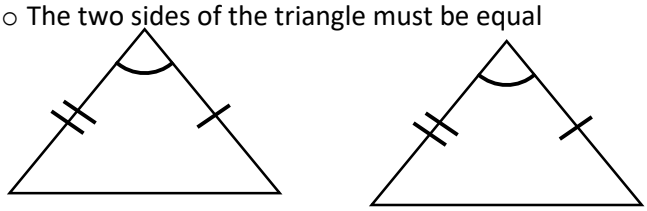
- Similarity can be worked out by the AAA (Angle – Angle – Angle) rule.
- **AAA (Angle – Angle – Angle) rule:** All the corresponding angles of the triangles must be equal.

3.2 Congruence

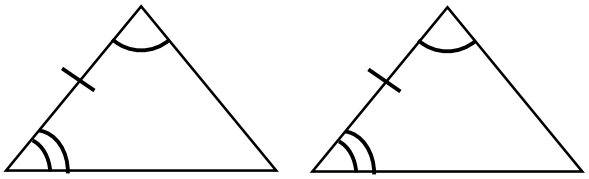
- **SSS (Side – Side – Side) rule:** All the three sides of the triangles must be equal



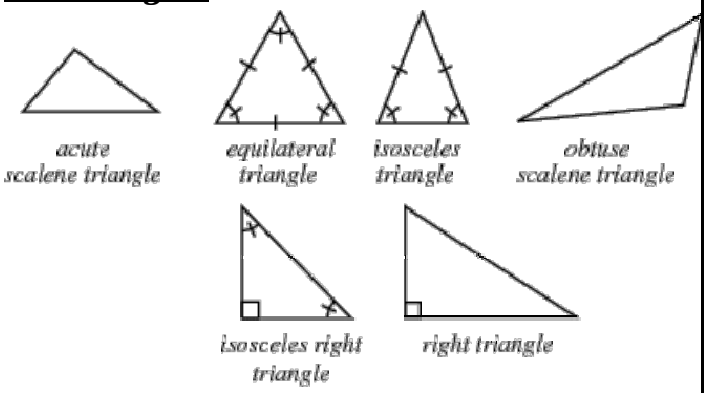
- **RHS (Right angle – Hypotenuse – Side) rule :**
  - There must two right-angled triangles
  - The length of the hypotenuses must be the same
  - One of the corresponding sides of each triangle must be the same
- **SAS (Side – Angle – Side) rule:**
  - There must be an angle and a side present
  - The angle of the adjacent sides must be equal
  - The two sides of the triangle must be equal



- **ASA (Angle – Side – Angle) rule:** The sides adjacent to the equal angles must be of the same length.

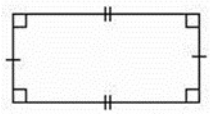


3.3 Triangles

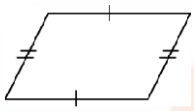


3.4 Quadrilaterals

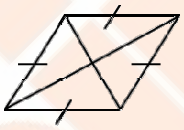
- **Rectangle:**
  - Opposite sides parallel/equal
  - all angles 90°
  - diagonals bisect each other



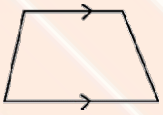
- **Parallelogram:**
  - Opposite sides parallel/equal
  - opposite angles equal
  - diagonals bisect each other



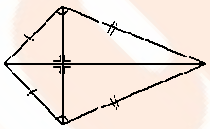
- **Rhombus:**
  - A parallelogram with all sides equal
  - opposite angles equal
  - diagonals bisect each other



- **Trapezium:**
  - One pair of sides parallel

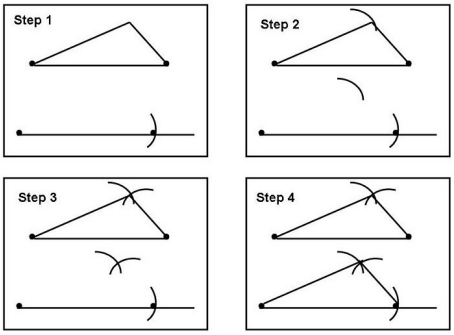


- **Kite:**
  - Two pairs of adjacent sides equal
  - diagonals are perpendicular to each other



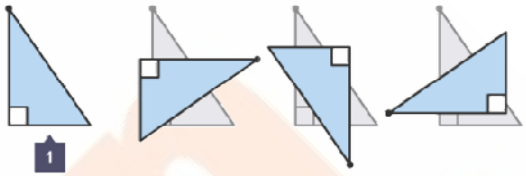
3.5 Construction

- **Constructing triangles:**



3.6 Symmetry

- **Line of symmetry**: Divides a two-dimensional shape into two congruent (identical) shapes
- **Plane of symmetry**: Divides a three-dimensional shape into two congruent solid shapes
- The number of times shape fits its outline during a complete revolution is called the order of **rotational symmetry**



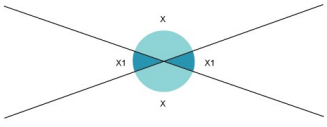
Shape	*Number of Lines of Symmetry*	*Rotational Symmetry Order*
Square	4	4
Rectangle	2	2
Parallelogram	0	2
Rhombus	2	2
Trapezium	0	1
Kite	1	1
Equilateral triangle	3	3
Regular hexagon	6	6

- **Properties of circles:**
  - Equal chords are equidistant from the centre
  - The perpendicular bisector of a chord passes through the centre
  - Tangents from an external point are equal in length

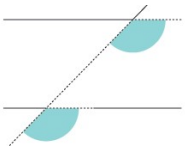
3.7 Polygons

- Sum of angles at a point =360°
- Angles on a straight line = 180°
- Sum of angles in a triangle =180°
- For regular polygon
  - External angles $= \frac{360°}{n}$
  - Internal angles $= 180° - \frac{360°}{n}$
- For irregular polygon:
  - Sum of exterior angles =360°
  - Sum of interior angles=180(n-2)

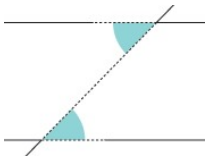
- Vertically opposite angles are equal



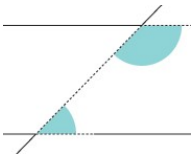
- Corresponding angles are equal



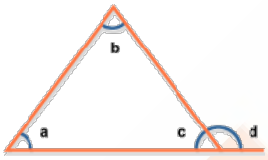
- Alternate angles



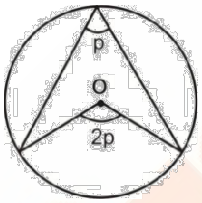
- Co-interior angles add up t0 180°



- Exterior angle=sum of interior opposite ∠

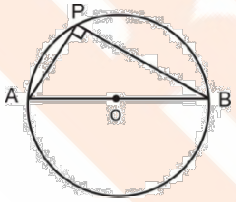


3.8 Circle Theorem

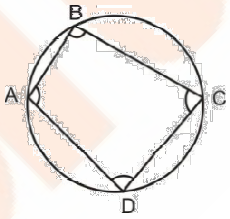


Angle at center = twice angle on circumference

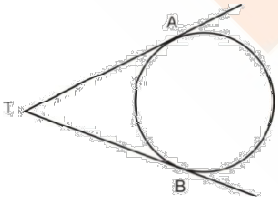
Angle subtended by same arc at circumference are equal



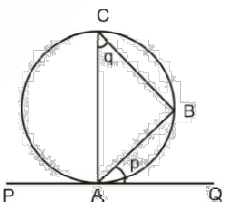
Angles in semicircleare 90°



Opposite angles in a cyclic quadrilateral = 180°



Tangents from one point are equal. ∠ between tangent and radius is 90°



Alternate segment theorem

4. MENSURATION

4.1 Area

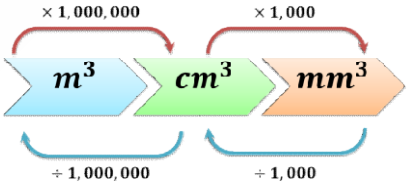
- Parallelogram $= b \times h$ OR $ab \sin\theta$
- Triangle$= \frac{1}{2} b \times h$
- Trapezium$= \frac{1}{2}(a+b)h$
- Circle$= \pi r^2$
- Sector$= \pi r^2 \times \frac{\theta}{360}$

4.2 Volume and Surface Area

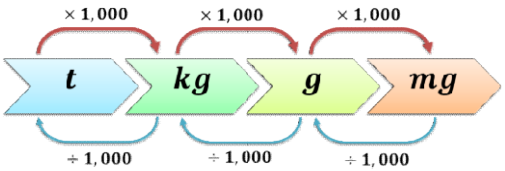
- Cuboid
  - $Surface\ area = 2lw + 2hl + 2hw$
  - $Volume = hlw$
- Cylinder
  - $Curved\ surface\ area = 2\pi rh$
  - $Volume = \pi r^2 h$
- Cone
  - $Curved\ surface\ area = \pi rl$
  - $Volume = \frac{1}{3}(\pi r^2 h)$
- Sphere
  - $Surface\ area = 4\pi r^2$
  - $Volume = \frac{4}{3}\pi r^3$
- Hemisphere
  - $Surface\ area = 2\pi r^2$
  - $Volume = \frac{2}{3}\pi r^3$

4.3 Units

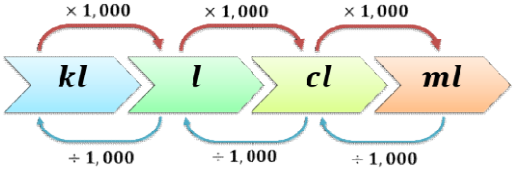
- **Volume:**



- **Mass:**



- **Capacity:**



- **Connecting volume and capacity:**
  - $1ml = 1cm^3$
  - $1kl = 1m^3$
- **Density** $= \frac{Mass}{Volume}$

5. Coordinate Geometry

5.1 Graphs

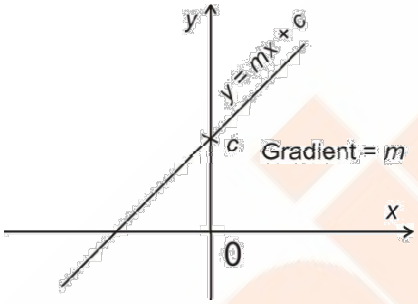
- **Gradient of a Straight Line:**

$$Gradient = \frac{y_2 - y_1}{x_2 - x_1}$$

- **Equation of Line:**

$$y = mx + c$$

  - Find the gradient, m
  - Find the y-intercept, c



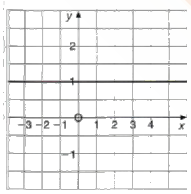
- **Midpoint of Graph:**

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

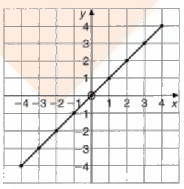
- **Length between two points:**

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

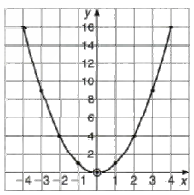
5.2 Sketching Graphs



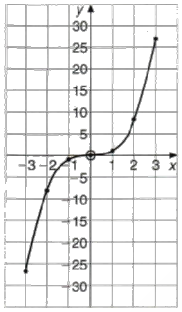
$f(x) = 1$



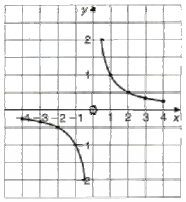
$f(x) = x$



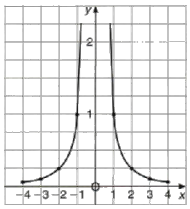
$f(x) = x^2$



$f(x) = x^3$



$f(x) = \frac{1}{x}$

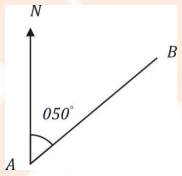


$f(x) = \frac{1}{x^2}$

6. Trigonometry

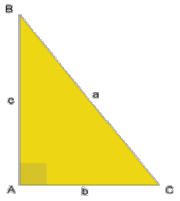
6.1 Bearings

- **The bearing of a point B from another point A is:**
  - An angle measured from the north at A.
  - In a clockwise direction.
  - Written as three-figure number (i.e. from 000 ° to 360°)
- e.g. The bearing of B from A is 050°

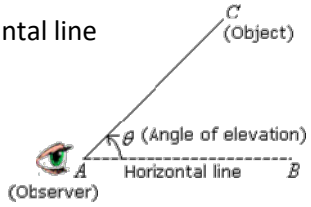


6.2 Pythagoras Theorem

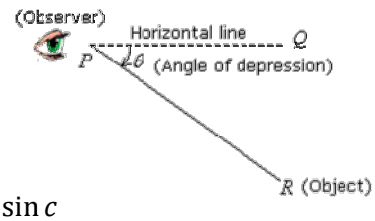
- To find hypotenuse
  - $a^2 + b^2 = c^2$
- To find one of the shorter sides
  - $a^2 = c^2 - b^2$
  - $b^2 = c^2 - a^2$



- **Angle of elevation:**
  - Angle above the horizontal line



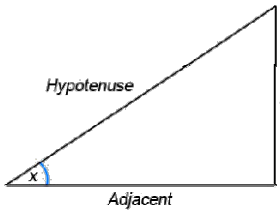
- **Angle of depression:**
  - Angle below the horizontal line.



- **Area of a triangle:** $\frac{1}{2}ab\sin c$

6.3 Ratios

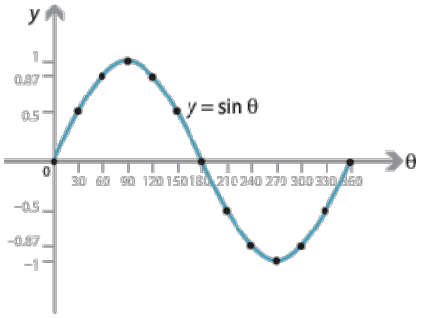
- **Right angled triangles:**
  - $\sin x = \frac{opposite}{hypotenuse}$
  - $\cos x = \frac{adjacent}{hypotenuse}$
  - $\tan x = \frac{opposite}{adjacent}$



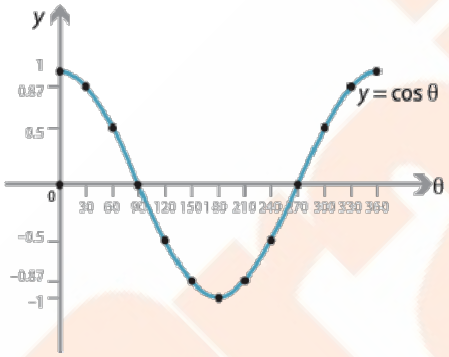
SOH
CAH
TOA

6.4 Graphs of simple trigonometric functions

- $\sin(x) = \sin(180 - x)$

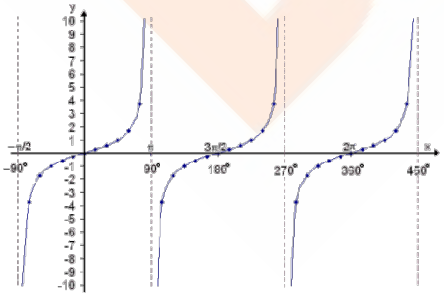


- $\cos(x) = \cos(360 - x)$



- Sine and cosine shifted by 90°
- Sine has x-intercepts at multiples 180°, and cosine at (90° + multiples of 180°)

- $\tan(x) = \tan(180^\circ + x)$



- Goes to infinity at 90°, 270°, 450°, …
- Has x-intercepts at multiples of 180°

6.5 Sine & Cosine Rules

- **Sine rule:**

$$\frac{a}{\sin a} = \frac{b}{\sin b} = \frac{c}{\sin c}$$

- **Cosine rule**
  - To find the angle given 3 sides

$$\cos a = \frac{b^2 + c^2 - a^2}{2bc}$$

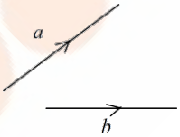
  - To find side given angle and two sides

$$a^2 = b^2 + c^2 - 2bc\cos a$$

7. Vectors & Transformations

7.1 Vectors

- **Vector quantity** has both magnitude and direction
  - E.g. Vectors a and b represented by the line segments, can be added using 'parallelogram rule' or 'nose-to-tail method'



- **Multiplication by a scalar:**
  - **Scalar quantity:** has a magnitude but no direction
  - The negative sign reverses the direction of the vector
- **Column vector:**
  - Top number = horizontal component
  - Bottom number = vertical component

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

- **Parallel vectors:**
  - Vectors are parallel if they have the same direction
  - In general, the vector $k\binom{a}{b}$ is parallel to $\binom{a}{b}$
- **Modulus of a vector:**
  - In general, if $x = \binom{m}{n}$, $|x| = \sqrt{(m^2 + n^2}$

7.2 Transformation

- **Reflection (M):**
  - When describing a reflection, the position of the mirror line is essential
- **Rotation (R):**
  - To describe a rotation, the centre of rotation, the angle of rotation and direction of rotation are required
  - A clockwise rotation is negative, and an anticlockwise rotation is positive
- **Translation (T):**
  - When describing a translation, it is necessary to give the translation vector

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

- **Enlargement (E):**
  - To describe an enlargement, state the scale factor, K and the centre of enlargement

$$Scale\ factor = \frac{length\ of\ \textbf{image}}{length\ of\ \textbf{object}}$$

$$Area\ of\ image = K^2 \times\ area\ of\ object$$

  - If K > 0, both object and image lie on same side of the centre of enlargement
  - If K < 0, object and image lie on opposite side of the centre of enlargement

8. Probability

- Probability is the study of chance, or the likelihood of an event happening

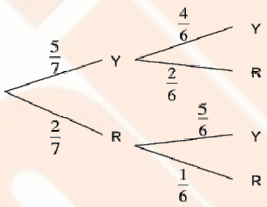
$$\text{Probability of an event} = \frac{number\ of\ \textbf{favourable}\ outcomes}{total\ \textbf{number}\ of\ outcomes}$$

- If probability = 0, event is impossible
- If probability =1, event is certain to happen
- All probabilities lie between 0 and 1

8.1 Events

Exclusive events:

- Two events are exclusive if they cannot occur at the same time
- **The OR Rule:**
  - For exclusive events A and B
  - P(A or B) = P(A) + P(B)

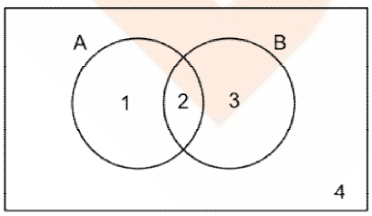


Independent events:

- Two events are independent if occurrence of one is unaffected by occurrence of other
- **The AND Rule:**
  - P(A and B) = P(A) × P(B)

8.2 Conditional Probability

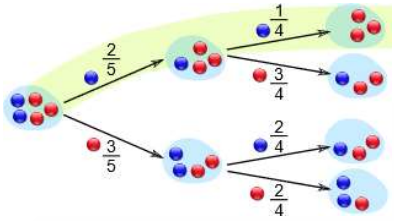
- Probability of an event (A), given that another (B) has already occurred [Symbol: P(A|B)]



P (A | B) is A *given* B

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{2}{2+3} = \frac{2}{5}$$

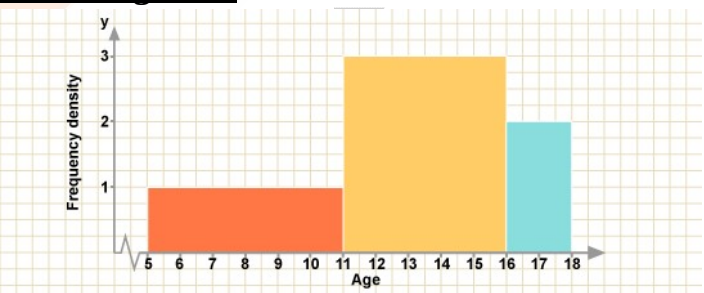
- Calculate using Venn diagram:
  - Construct the Venn diagram, using sample space of both events
  - P(A|B) = P(A ∩ B) / P(B)
- Calculate using tree diagrams:



  - Construct tree diagram.
  - Write the outcomes of the first event, connecting them to each outcome of the second event. Write probability on top of each event's line
  - Multiply probabilities on the lines to the required outcome
  - Note: The probabilities reduce with each step if objects are replaced
- Calculate using two-way tables:
  - Column and row headers are the sample space of the two events
  - Fill in each cell with the correct number of outcomes
  - Take the required number from the table and divide by the sum of all values in the row/column of the condition provided.
- Remember: P(A|B) and P(B|A) are not the same

9. Statistics

9.1 Histograms



- **Histogram**: Displays frequency of either continuous or grouped discrete data in the form of bars
- Bars are joined together and may be of varying width
- Frequency of the data is represented by the area of the bar and not the height
- When class intervals are different, area of the bar represents the frequency, not the height
- Frequency density plotted on y-axis, not frequency
- **Class width** = Interval
- **Frequency density** = Height

$$Frequency = Class\ width \times Frequency\ density$$

9.2 Averages

- **Mean:**

$$\frac{Sum\ of\ values}{number\ of\ values}$$

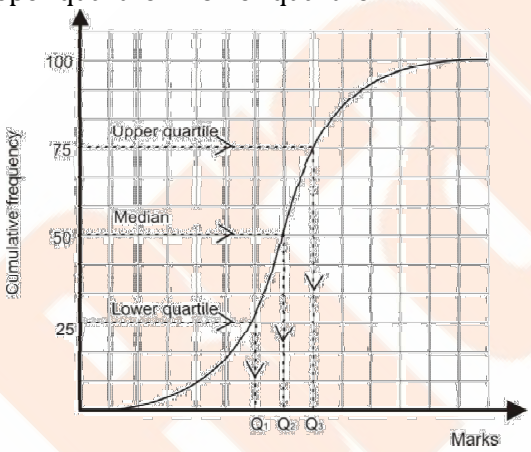
- **Median:**
  - The middle value - when the data has been written in ascending or descending order
  - Odd no. of values $\frac{5+1}{2} = 3rd\ value$
  - Even no. of values $\frac{6+1}{2} = 3.5th\ value$ (add two values divide by 2)
- **Mode:**
  - Most frequently occurring value
- **Range:**
  - Difference between highest and lowest values

- **Estimated mean of grouped data:**
  - Work out midpoints of each group and multiply by frequency
  - Divide by number of values

9.3 Cumulative Frequency

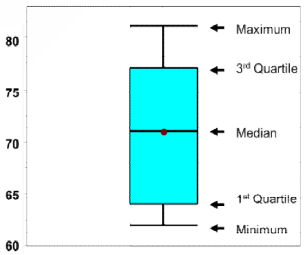
- Cumulative frequency is the total frequency up to a given point
- Inter-quartile range

$= \text{upper quartile} - \text{lower quartile}$

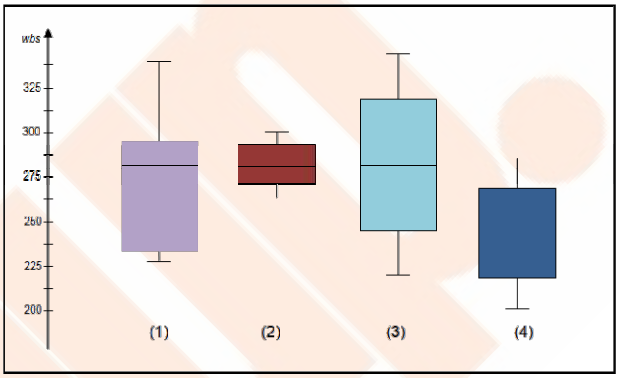


9.4 Box-and-whisker plots

- Construction
  - Find median and two quartiles
  - Draw three lines of equal width along these values
  - Complete the boxes
  - Draw 'whiskers' extending from the box to the maximum and minimum values, draw two more lines at the ends



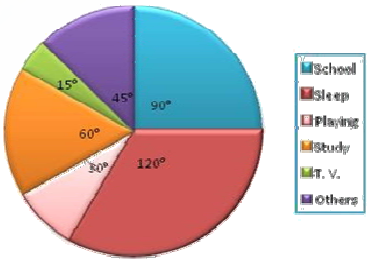
- Interpretation:
  - Median, quartiles and extreme values can be found by reading on the scale on the y-axis
  - Short boxes mean low IQR and vice versa (2), (3)
  - Long whiskers mean a lot of extreme values and vice versa (1)
  - Difference in position of boxes represents if data in one set is overall higher or lower than another data set. (3) and (4)
  - Variation in lengths of different sections and position of median show how evenly the data is spread, compared to other data sets (1)



9.5 Pie Charts

- Sectors represent data, and these sectors form a circle.
- Angle of a sector:

$$\theta = \frac{Number\ of\ an\ item}{Total\ number\ of\ items} \times 360^{\circ}$$



- Sum of angles in a pie chart is 360°

9.6 Stem and Leaf diagrams

- Stem-and-Leaf diagram is a quick way of summarizing a range of data.
- There is a column known as the stem, contains which contains unique elements of data formed by removing last digits of the data.
- Keys are used in this diagram

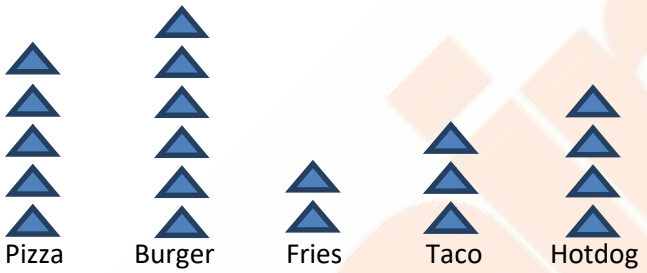
stem	leaf
0	1, 1, 2, 2, 3, 4, 4, 4, 4, 5, 8
1	0, 0, 0, 1, 1, 3, 7, 9
2	5, 5, 7, 7, 8, 8, 9, 9
3	0, 1, 1, 1, 2, 2, 2, 4, 5
4	0, 4, 8, 9
5	2, 6, 7, 7, 8
6	3, 6

Key: 6|3 = 63 years old

9.7 Pictograms

- Data is represented in pictures
- A key is given to represent the value of a picture.

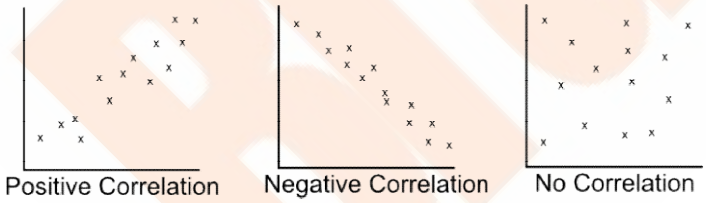
E.g. ▲ = 5 people



Favorite Fast Food of 100 Children

9.8 Scatter Diagrams

- Displays the correlation between two sets of data
- May have positive, negative or no correlation



- Line of best fit drawn through points that has an equal number of points on each side to show the trend